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CONTRIBUTION TO RESEARCH OF DROUGHTS IN VOJVODINA PROVINCE

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Abstract: Complete stochastic analysis of droughts, which are defined here as the upper extremes of the rainless periods is carried out in this paper. All important components of drought period processes such as their total number in a given time interval-growing season, their average number, their duration, the longest rainless period and its time of occurrence are obtained by analysis for meteorological station Rimski Šančevi, Vojvodina Province. Comparisons of drought analysis for two different periods for the same meteorological station are carried out: Period A: 1949-1985 and Period B: 1986-2010. Good agreement is found between the theoretical and empirical distribution functions for all analyzed components of the process for both analysed periods. On the basis of obtained results it is possible to conclude that there are in average more than two droughts during whole growing season for period B and less than two for period A. The average period without rain exceeding 15 days is 23.8 days for period A and 24.5 days for period B. Simple diagrams of return periods of drought durations are also obtained. The period of second half of August and first half of September is the part of the growing season that has the highest probability of having the longest drought for both analyzed periods. Comparison of drought analysis revealed that the droughts in Vojvodina Province were common and also bigger for the period B, which means after 1986.

Key words: Drought, Extremes, Rainless period, Distribution functions, Statistical tests, Return periods of droughts

1. INTRODUCTION

The Vojvodina Province is predominantly an agricultural region, which is characterized by a continental climate with accentuated annual variability of air temperatures, as well as the others climatic elements. There is a serious lack of water in the periods of droughts (Rajić & Rajić, 2005). These past years, droughts have been a common occurrence in the Vojvodina Province. After the dry 1980's, also 1990's (Palfai & Darnai, 1990) and 2000's were a drought years (Rajić et al., 2006). In order to determine dry years, following criteria have been used: the hydrotermic coefficient by Seljaninov, the index of aridity by De Martone, the rain factor by Lang and the coefficient and ratio between precipitation and evapotranspiration for the growing period and for whole year (Rajić et al., 2006). These analyses and analyses of precipitation as well as other climatic elements revealed that the dry years were more common for the period after 1981 in Vojvodina Province. They were mainly a consequence of the shortage in rainfalls and/or their unfavorable time distribution. Drought with its characteristics is a phenomenon with very complex properties. It can be determined by either one or more component such as: periods of no rainfall, insufficient rainfalls, high air temperature (Hobai, 2009), low relative air humidity, high evapotranspiration, some dryness index (Dragota & Kucsicsa, 2011) etc. Regional frequency analysis and time series simulation by means of nearest-neighbour resampling were studied to estimate a large quantile of distribution of the annual maximum precipitation deficit (Beersma & Buishand, 2007). Using memory term they found an enhanced positive correlation between past and future precipitation deficits during extremely dry summers. Derivation of the probability distribution of droughts episodes considering both drought duration and accumulated deficit (or intensity) as well as of the ensuing return period is investigated by Cancelliere & Salas (2010).

Discrete Autoregressive Moving Average (DARMA) model is proposed to better model drought length. In this paper the concept of return period was applied to characterize the severity of extreme droughts. In order to survey droughts a standard precipitation index (SPI) was determined (Moradi et al., 2011). Also, they were combined RUN theory and Markov Chain for forecasting the conditions of long term droughts in terms of number and drought duration. Mishra & Singh (2011) pointed out that there were significant improvements in modelling droughts over the past three decades. In this article they give a review of the methods used for modelling different components of droughts, which could be useful for different sectors dealing with water resources directly or indirectly. Drought hazards, and the ability to mitigate them with advance warning, offer potentially valuable applications of climate forecast products (Steinemann, 2006). Although the many definitions and indicators of droughts exist, the fact is that the main cause of drought is insufficiency of rainfall. The aim of this paper is not to give the best definition of drought; it is rather an attempt to compare drought analysis for two different periods at the same meteorological station. The analysis is carried out for the growing season, because of its high importance for agriculture. Droughts are defined here as the upper extremes of dry weather intervals. The analysis included all available data on extreme dry weather intervals, which are defined as the upper extremes of intervals of no rainfall longer than 15 days. Roughly speaking, the reference value Y_r is 15 days for field crops, 25 days for forage crops, and 30 days for orchards and vineyards. About 90% of agricultural land in the analyzed region is used for field crops. Because of that, the reference level Y_r is set at 15 days. Available data of rainless periods, lasting more than 15 days, are analyzed for two historical periods: period A: 1949-1985 and period B: 1986-2010. Period A is already analyzed in previous work by author of this paper (Berić et al., 1990) and results are shown here too. Drought analysis and comparison of obtained results by use of stochastic method is an effective way how to describe drought phenomenon that have occurred in Vojvodina Province.

2. METHOD OF DROUGHT ANALYSIS

The statistics of extreme have played an important role in engineering practice for water resources design and management (Katz et al., 2002). The method is used here is recommended for the analysis of extreme dry weather intervals, which are also synonymously called extreme rainless periods, or droughts in this paper. The method attempts to develop a general stochastic model of extreme rainless intervals and uses the data on rainless periods above a given reference value, Y_r. All important components of the process, such as drought duration, time of occurrence, number of droughts in a given time interval [0,t], the longest drought in a given time interval [0,t], and its time of occurrence, are taken into consideration. Droughts are defined here as the upper extremes of dry weather intervals and are treated as a random number of random variables in an interval of time [0,t]. The method is based on the assumption that droughts are independent, identically distributed random variables and that their occurrence is subject to the Poisson probability law. An application of the method is shown for the part [0,t] of the year which is equal to the growing season (1 April - 30 September), because it is of prime importance for agriculture, although the method can be applied for the entire year or part of it. Data on rainless intervals also included days with a precipitation depth of 3 mm or less on one day, and days with a precipitation depth of 5 mm or less on three consecutive days with the condition that each of these days did not have more than 3 mm of rain.

Regarding the reference value Y_r, two groups of rainless intervals are obtained:

a) $Y_m > Y_r$, with $(Y_m - Y_r) > 0$
b) $Y_m \leq Y_r$

Values $Y_m > Y_r$ as the upper extremes of rainless periods are especially considered in this paper. They are designated as X_v and called droughts,
where $v = 1,2, ...,$ and $v \leq m$

Each drought event is composed of the following defining descriptive parameters:

(1) drought duration, X_v;
(2) time of the beginning of a drought, $\tau_b\ (v)$;
(3) time of the end of a drought, $\tau_e\ (v)$;
(4) time of a drought occurrence, $\tau(v)$, defined here as

$$\tau(v)=\frac{1}{2}\left[\tau_b(v)+\tau_e(v)\right]$$

(5) order number of a drought, v, for a given time interval [0,t], for a particular growing season, where $v = 1,2, ...$.

Considering the entire process of droughts, three additional magnitudes enter the analysis:

(6) total number of droughts, k, within the time interval [0,t], where $k = 0, 1, 2, ...$;
(7) the longest (largest) drought within a time interval [0,t]

$X(t) = \sup X_v$ and $\tau(v) \leq t$

(8) time of occurrence, $\tau(t)$, of the longest (largest) drought, within time interval [0,t].

The presented method involves also another random variable, Z_v, in an interval of time [0,t] defined as:

$$Z_v = X_v - Y_r,$$

or, here,

$$Z_v = X_v - 15,$$

where Z_v and X_v are measured in days.

According to the nature of drought phenomenon, it is obvious that their total number in time interval [0,t], as well as their duration and time of occurrence, are random variables. In this way the stochastic process of extreme rainless periods may be completely described. Each component of the process can be analyzed by the use of some mathematical description/formulae, here distribution functions. The simple statistical tests (Pearson χ^2-test and Kolmogorov-Smirnov test) are applied for evaluation of the agreement between theoretical and empirical distribution functions for all analyzed components of the process. Further explanations concerning theoretical background can be found in previous works (Zelenhasić, 1970; Todorović & Zelenhasić, 1970; Todorović & Woolhiser, 1972; Berić et al., 1990).

3. RESULTS

Data on dry weather intervals are used separately for each period in order to determine the probabilistic structure of droughts at the meteorological station Rimski Šančevi (Table 1).

Table 1. Data from meteorological station Rimski Šančevi used for analysis of droughts

Analyzed period	*Data period*	*No. of years without droughts*	*Total No. of droughts*	*Longest drought in days*
Period A	1949-1985	3	70	61
Period B	1986-2010	0	55	52

A hypothesis stating that droughts are independent and identically distributed random variables was verified and accepted after applying the run test at a 5% significance level. Serial correlation coefficients were also computed for the drought time series and significance test at a 5% probability level confirmed that dry weather intervals are independent. The degree of correlation among certain data is practically negligible.

3.1. Distribution of the number of droughts

The distribution of the number of droughts at a given location is an important component of the method. Following the previous work (Zelenhasić, 1970; Berić et al., 1990), it is expected that the number of droughts in [0,t] at meteorological station Rimski Šančevi will also be distributed according to the Poisson probability law. That was proved to be case for both analyzed period and also for all six different periods of the growing season. Each of these periods starts with April 1 and lasting 30, 61, 91, 122, 153 and 183 days respectively. In order to estimate function λ(t), the distribution of the number of droughts is analyzed also for six different parts of the growing season (Table 2). The function λ(t) enters the Poisson probability law,

$$P\left(E_k^t\right) = \frac{\left[\lambda_1(t)\right]^k}{k!} e^{-\lambda_1(t)} \quad (1)$$

where

E_k^t - the event that exactly k droughts occur in [0,t]

and where

k = 0,1,2, … and

[0,t] = [1 April - 30 September] = 183 days

$\lambda_1(t)$ - the mean number of droughts in a time interval [0,t]

$$e^{-\lambda_1(t)} = e^{-1.892} = 0.1508 \text{ -Period A}$$

$$e^{-\lambda_1(t)} = e^{-2.20} = 0.1108 \text{ -Period B}$$

For the meteorological station Rimski Šančevi this equation is computed as:

$$\text{Period A: } P\left(E_k^t\right) = 0.1508 \cdot \frac{1.892^k}{k!} \quad (2)$$

$$\text{Period B: } P\left(E_k^t\right) = 0.1108 \cdot \frac{2.20^k}{k!} \quad (3)$$

Table 2. The average number of droughts for meteorological station Rimski Šančevi

Analyzed period	*Time intervals*					
	April 1 – April 30	*April 1 – May 31*	*April 1 – June 30*	*April 1 – July 31*	*April 1 – August 31*	*April 1 – Septem. 30*
Period A	0.216	0.486	0.622	0.973	1.432	1.892
Period B	0.320	0.600	0.920	1.280	1.880	2.200

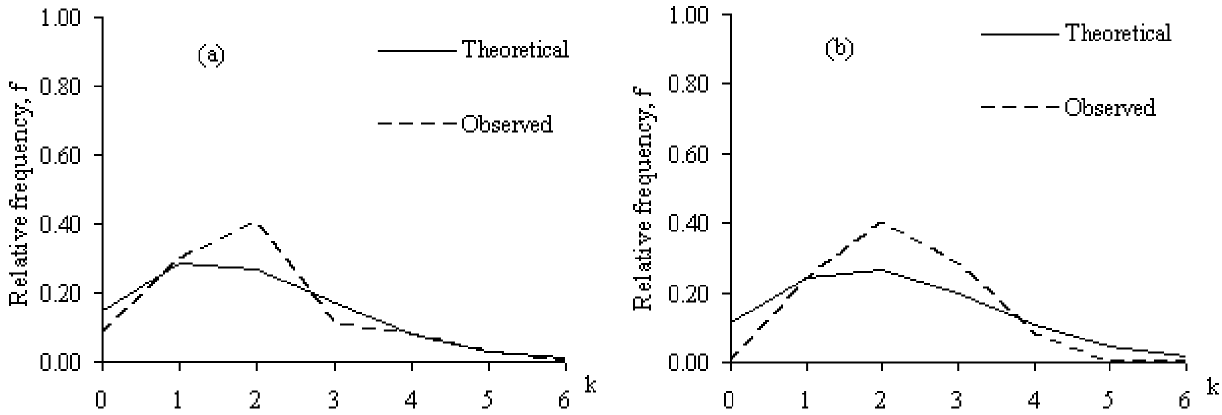

Figure 1. The observed and corresponding theoretical (Poissonian) distributions of the number of droughts for the growing season (183 days) at Rimski Šančevi (a) Period A: 1949-1985 and (b) Period B: 1986-2010

Tests of the conformance of the theoretical distribution to the observed distribution at 5% significance level indicate a good agreement for both analyzed periods. As example, the observed and corresponding theoretical (Poissonian) distributions for whole growing season (183 days) are shown on figure 1.

3.2. Distribution of drought durations

The first step here is to determine the distribution of the random variable Z_v and then to analyze drought duration as variable X_v. The time interval [0,t] was set equal to the growing season. The corresponding theoretical exponential distribution function is determined as:

$$B(Z)=1-e^{-\lambda_2 \cdot Z}; \quad Z \geq 0 \qquad (4)$$

where

λ_2 - parameter which is estimated as $\lambda_2 = \frac{1}{\overline{Z}}$

Mean and standard deviation of the random variable Z_v:

Period A: $\overline{Z}=8.8\,days, \qquad \sigma(Z)=8.2\,days$

Period B: $\overline{Z}=9.5\,days, \qquad \sigma(Z)=9.2\,days$

The maximum observed values of Z_v are:
Period A:
max Z_{rec} = 46 days (max X_{rec} = 46+15 = 61days)
Period B:
max Z_{rec} = 37 days (max X_{rec} = 37+15 = 52 days)
Theoretical distributions functions are:

Period A: $B(Z)=1-e^{-0.1140 \cdot Z}; \quad Z \geq 0 \qquad (5)$

Period B: $B(Z)=1-e^{-0.1053 \cdot Z}; \quad Z \geq 0 \qquad (6)$

Observed and corresponding theoretical distribution functions of the random variable Z_v are shown in figure 2. It can be seen that the exponential distributions gives a good fit to the observed distribution. The Kolmogorov – Smirnov test confirmed the good agreement between the two distribution functions for both analyzed periods.

3.3. Distribution of the largest drought

The distribution of the largest value of the random variable Z_v in a time interval [0,t] is determined in this subsection and then the result for X_v is obtained. The theoretical distribution function of the longest drought is determined as double exponential distribution function:

$$F(Z/t)=e^{-\lambda_1(t) \cdot e^{-\lambda_2 \cdot Z}} \quad Z \geq 0 \qquad (7)$$

where:
$\lambda_1(t)$ - the mean number of droughts during the growing season
λ_2 - parameter of distribution of drought durations during the growing season
The average value and standard deviation of the largest drought are:

Period A: max. $\overline{Z}$ =12.4 days; $\sigma(\max.Z)$=9.9 days

Period B: max. $\overline{Z}$ =15.1 days; $\sigma(\max.Z)$=10.8 days

In (7), the time interval [0,t] can be any part of the growing season or the whole season, which depends upon the value of the deterministic function $\lambda(t)$. The case of greatest appeal in practice is the time interval [0,t] of the whole growing season. In that case:

Period A: $F(Z/t)=e^{-1.892 \cdot e^{-0.1140 \cdot Z}} \qquad (8)$

Period B: $F(Z/t)=e^{-2.20 \cdot e^{-0.1053 \cdot Z}} \qquad (9)$

The graphs of the theoretical and observed distribution functions of the largest value of Z_v for the given meteorological station are depicted in figure 3. Both the Kolmogorov – Smirnov and the chi-square goodness of fit tests have shown good agreement between the two distribution functions given in figure 3.

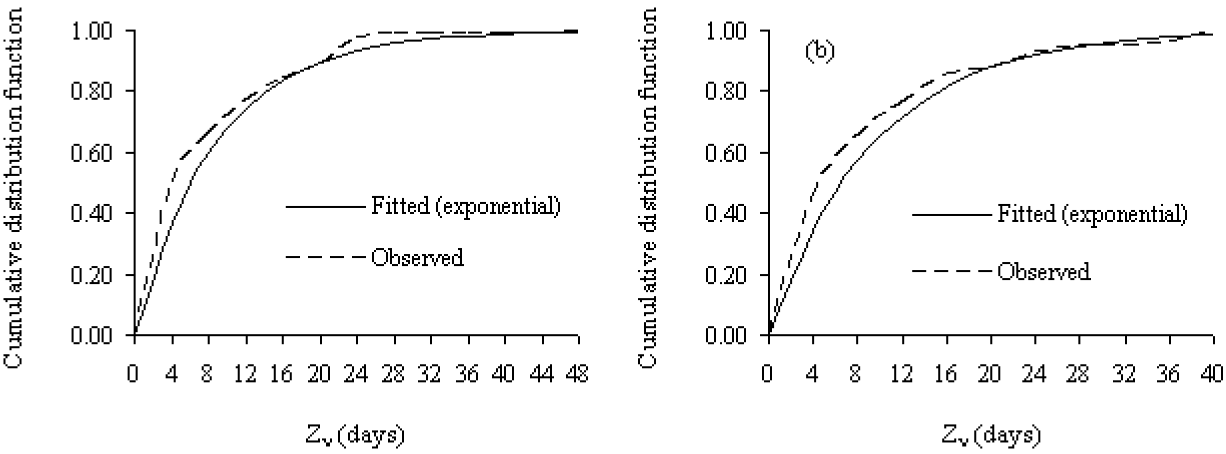

Figure 2. The theoretical and observed distribution function of random variable Z_v for the growing season at Rimski Šančevi (a) Period A: 1949-1985 and (b) Period B: 1986-2010

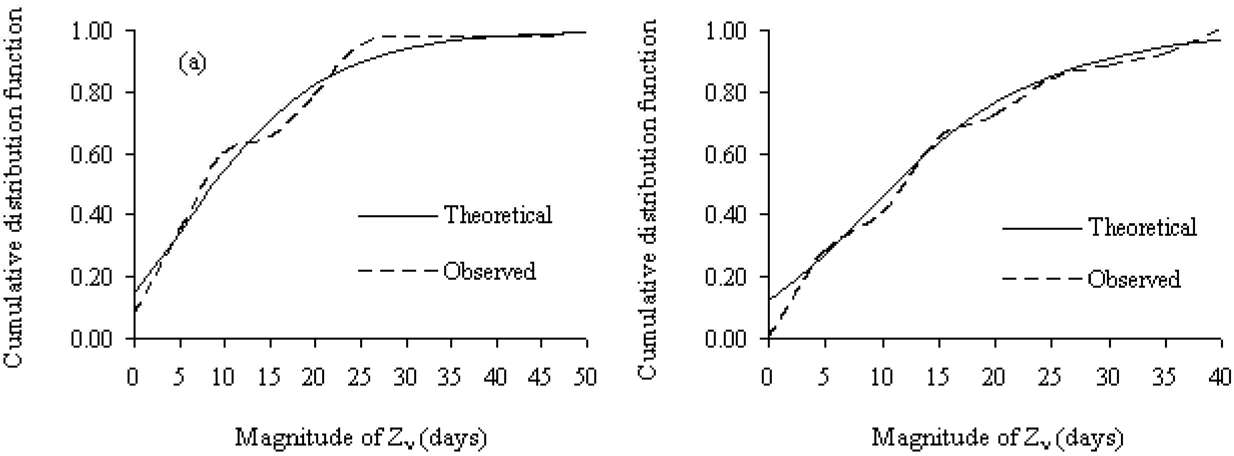

Figure 3. The theoretical and observed distribution function of random variable Z_v for the growing season at Rimski Šančevi (a) Period A: 1949-1985 and (b) Period B: 1986-2010

Practical value of the theoretical distribution function of the longest drought is possibility to estimate numerical values X_v of droughts for different return periods. On the basis of equations (8) and (9) and stochastic variable

$$Z_v = X_v - Y_r$$

or, here,

$$Z_v = X_v - 15$$

as well as the expression for the return period T_r of stochastic variable Z_v

$$T_r = \frac{1}{1 - F(Z/t)} \qquad (10)$$

a relationship $T_r = f(X_v)$ is determined and is shown in figure 4. This simple diagram can help to determine the return period for every drought observed at meteorological station Rimski Šančevi. It is of great importance for a practising engineer.

3.4. Distribution of the time of occurrence of the longest drought

In practice, it is important to know which part of the growing season has the highest probability of the longest drought occurring in it. The longest dry weather intervals in the growing season are plotted as impulse functions in the middle of their time intervals with the magnitude equal to their durations. The results of Todorović & Woolhiser (1972) for the time when the supremum of a random variables occurs, are also used here:

$$F\{\tau(t) \le u\} = e^{-\lambda_1(t)} + \frac{\lambda(u)}{\lambda_1(t)}\left[1 - e^{-\lambda_1(t)}\right] \qquad (11)$$

where

$\lambda_1(t)$ - the mean number of droughts in the growing season u – numerical value which is taken by random variable τ(t), and where $0 \le u \le t$ and u is given in days

$\lambda(u)$ - value from function $\lambda(t)$

τ(t) – moment in time interval [0,t] when the longest extreme rainless period occurred

For meteorological station Rimski Šančevi equation becomes

Period A:

$$F\{\tau(t)\le u\}=0.1508+0.4489\lambda(u) \qquad (12)$$

Period B:

$$F\{\tau(t)\le u\}=0.1108+0.4042\cdot\lambda(u) \qquad (13)$$

if [0,t] is equal to the growing season.

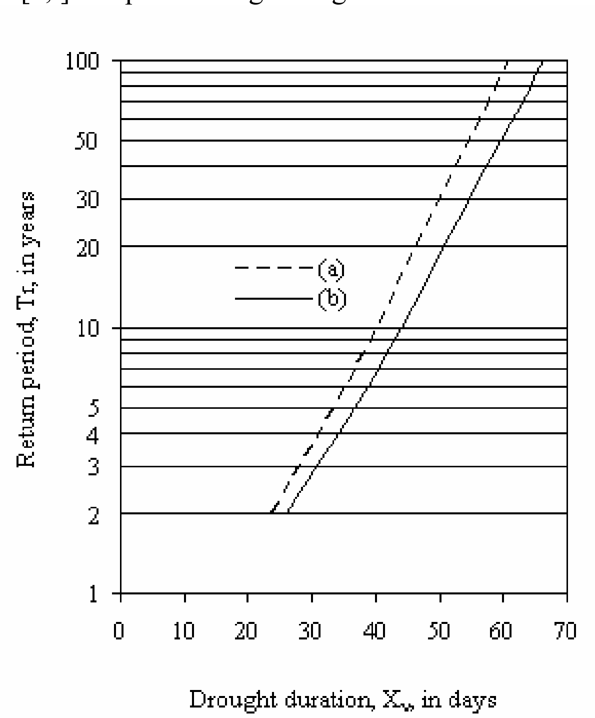

Figure 4. Return periods of drought durations occurring in the growing season for Rimski Šančevi
(a) Period A: 1949-1985 and (b) Period B: 1986-2010

The Kolmogorov - Smirnov goodness of fit test has shown a good agreement between the two distribution functions. Using Equation (11) for both analyzed periods it is possible to indicate the part of the growing season with the highest probability of having the longest drought. The results of such analysis show that it is a part of growing season referring to the period of second half of August and first half of September, for both analyzed periods.

4. CONCLUSIONS

As was already mentioned, a complete stochastic analysis of droughts was carried out separately for each of two different periods at meteorological station Rimski Šančevi, Vojvodina Province. Good agreement is found between the theoretical and empirical distribution functions for all analyzed components of the process for both analyzed periods. On the basis of obtained results it is possible to conclude that average number of droughts for meteorological station Rimski Šančevi is bigger for the second period (period B) for all different periods of the growing season. There are in average more than two droughts during whole growing season for period B and less than two for period A.

The average period without rain exceeding 15 days is 23.8 days for period A and it exactly corresponds to 2-year return period. For period B average values is 24.5 days and it corresponds to return period less than 2-year.

The return period for every drought can be determined according to distribution functions of the longest extreme rainless periods, derived for each analyzed period. Using Equations (12) and (13), it is easy to see that the period of second half of August and first half of September is the part of the growing season that has the highest probability of having the longest drought for both analyzed periods.

Comparison of drought analysis for two different periods at the same meteorological station, revealed that the droughts, defined here as upper extremes of rainless period, were common and also bigger for the period B, which means after 1986. Also, it could be concluded that applied method in this paper is very informative and gives comprehensive information.

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